A RADIATOR SYSTEM WITH CONICAL HEAT-REMOVAL PINS

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A solution is presented for the problem of calculating the heat-removal capacity and optimization of a system of radially divergent solid and hollow conical pins radiating into space.

A number of reports have been published on the possibility of intensifying the removal of heat from heated bodies in a vacuum by means of flat and annular radiator fins [1–4]. However, in those cases in which the surface of the body to be cooled resembles a polyhedron or a sphere (for example, the outer shell of a low-power thermal-emission or thermoelectric generator [5]), the heat-removal radiator elements need not be fins, but may be pins. Calculating the efficiency and optimization of such pins demands consideration of the mutual radiative interaction (separate pins of various shapes are examined in [6] and [7]).

Let us consider the problem of calculating and optimizing a radiator system in which the radially divergent conical pins radiating into space are situated on an isothermal sphere to be cooled (Fig. 1a).

Let us consider the case in which:

1) the radius of the sphere to be cooled—and through whose center the axes of the pins pass—is determined from the condition that the bases of the two closest pins come into contact on the surface of the sphere

$$r_0 = \frac{L}{\left[\dfrac{\sin\left(\dfrac{\alpha}{2}+\dfrac{\gamma_{\min}}{2}\right)}{\sin\dfrac{\alpha}{2}} - 1\right]}; \tag{1}$$

2) the pins are situated so that the points at which their axes intersect with the surface of the sphere represent the apices of a regular polyhedron inscribed within the sphere.

We will treat the problem under the following assumptions:

1. The surface of the body to be cooled that is free of pins is small in comparison with the surface of the pins and we may therefore neglect the mutual irradiation of the pins and the body to be cooled, taking into consideration only the mutual irradiation of the pins.

2. The temperature in the cross section of the pins is constant, and the pin surfaces are gray diffusion radiators.

3. The transfer of heat between any two pins of the system is equal to the transfer of heat between the longitudinal sections of these pins in planes perpendicular to the plane in which the axes of the pins lie (Fig. 1b). In calculating the mutual irradiation of the pins, we can replace the body to be cooled by a polyhedron, and the bases of the pins may be assumed to be planes (Fig. 1c).

4. The surrounding space is a black body with zero temperature.

With consideration of the foregoing, the law of thermal radiation and the equation of heat conduction along the pin will be valid in the following form:

$$Q = -\lambda \frac{dT}{dx} \pi (L-x)^2 \operatorname{tg}^2 \frac{\alpha}{2}, \tag{2}$$

$$dQ = -[E^*_{\text{eff}}(x) - \Sigma E^*_{\text{inc}}(x)]\, 2\pi (L-x) \operatorname{tg}\frac{\alpha}{2}\, dx. \tag{3}$$

Since Q = 0 and x = L, Eqs. (2) and (3) make it possible to derive the following expression for the determination of the temperature distribution along the pin:

$$\frac{dT}{dx} + \frac{2}{\lambda \operatorname{tg}\dfrac{\alpha}{2}(L-x)^2} \times \int_x^L [E^*_{\text{eff}}(x) - \Sigma E^*_{\text{inc}}(x)](L-x)\, dx = 0, \tag{4}$$

where

$$E^*_{\text{eff}}(x) = \varepsilon\sigma T^4 + (1-\varepsilon)\Sigma E^*_{\text{inc}}(x). \tag{5}$$

The boundary conditions for (4) are

$$T = T_0 \text{ when } x = 0. \tag{6}$$

Let us find the expression for $\Sigma E^*_{\text{inc}}(x)$ and for this we will initially determine the quantity $E_{\text{inc}\,z}(x)$ for the two pins positioned at an angle γ (Fig. 1b).

We will assume that the density of the incident radiation is independent of the coordinate y (see Fig. 1b) and is equal for each value of x to the mean magnitude along the strip $2(L-x)\operatorname{tg}(\alpha/2)$. It is then not difficult to demonstrate that the expression for the determination of the density of the radiation incident on the pin under consideration from all of the adjacent pins will have the form

$$E_{\text{inc}\,z}(x) = \frac{1}{2\pi}\int_0^L \left(E_{\text{eff}}(z)(z+r_0)(x+r_0)\sin^2\gamma\psi\, dz\right) \times \left((L-x)\left[(z+r_0)^2 + (x+r_0)^2 - 2(x+r_0)(z+r_0)\cos\gamma\right]^{3/2}\right)^{-1} \tag{7}$$

where

$$\psi = [(L-z)+(L-x)] \operatorname{arctg} \times \frac{\left[(L-z)\operatorname{tg}\dfrac{\alpha}{2} + (L-x)\operatorname{tg}\dfrac{\alpha}{2}\right]}{[(z+r_0)^2+(x+r_0)^2 - 2(x+r_0)(z+r_0)\cos\gamma]^{1/2}} -$$

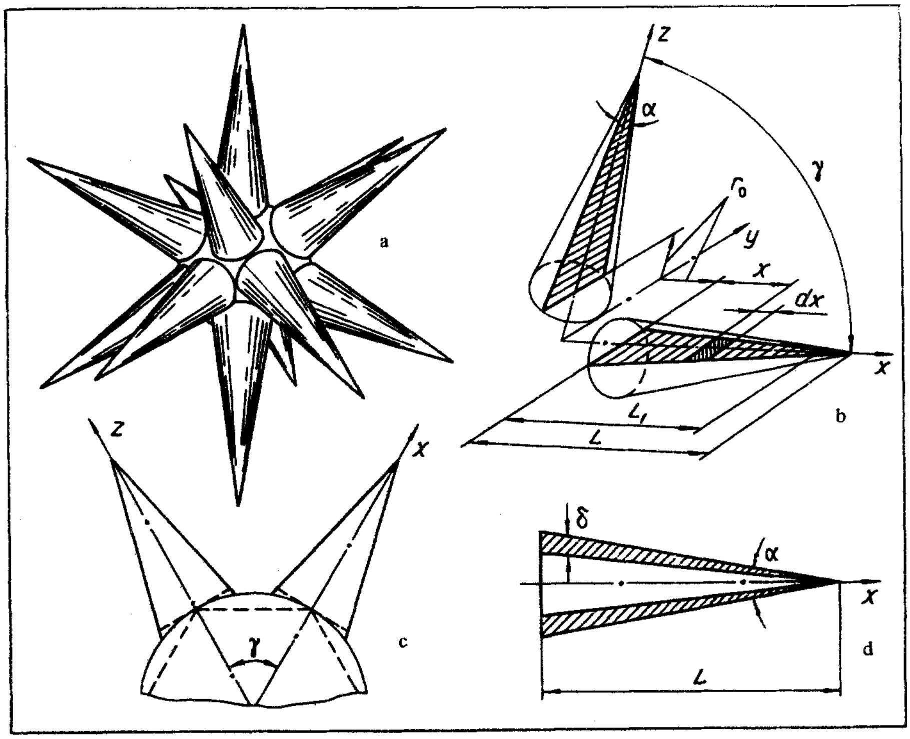

Fig. 1. Radiating system with conical heat-removal pins: a) over-all view; b) and c) calculation of radiative heat exchange between pins; d) hollow conical pin.

$$-[(L-z)-(L-x)]\,\mathrm{arctg}\times$$

$$\times\frac{\left[(L-z)\,\mathrm{tg}\,\frac{\alpha}{2}-(L-x)\,\mathrm{tg}\,\frac{\alpha}{2}\right]}{[(z+r_0)^2+(x+r_0)^2-2(x+r_0)(z+r_0)\cos\gamma]^{1/2}};$$

$$E_{\text{eff}}(z)=\varepsilon\sigma T^4+(1-\varepsilon)E_{\text{inc}\,x}(z). \quad (8)$$

The incident and effective radiations averaged over the perimeter at section x will be determined from the relationships

$$E^*_{\text{inc}\,z}(x)=\frac{1}{\pi}E_{\text{inc}\,z}(x), \quad (9)$$

$$E^*_{\text{eff}}(x)=\varepsilon\sigma T^4+(1-\varepsilon)E^*_{\text{inc}\,z}(x). \quad (10)$$

If there is radiant interaction between pin under consideration and a number of other pins, in this event we will have

$$\Sigma E^*_{\text{inc}}(x)=$$

$$\frac{1}{\pi}[E_{\text{inc}\,z}(x)+E_{\text{inc}\,\varphi}(x)+E_{\text{inc}\,\xi}(x)+\ldots], \quad (11)$$

where the subscripts z, φ, ξ, etc., denote the longitudinal coordinates of the pins with which the pin under consideration is in radiant interaction. $E_{\text{inc}\,\varphi}(x)$, $E_{\text{inc}\,\xi}(x)$, etc., are determined from the same relationship (7), the only difference being that the angle between the two pins under consideration must be substituted for the angle γ, and that, instead of $E_{\text{eff}}(z)$, we must substitute $E^*_{\text{eff}}(z)$, $E^*_{\text{eff}}(\varphi)$, $E^*_{\text{eff}}(\xi)$, etc., calculated from expression (5) (when $x = z = \varphi = \xi$; $E^*_{\text{eff}}(x) = E^*_{\text{eff}}(z) = E^*_{\text{eff}}(\varphi) = E^*_{\text{eff}}(\xi)$). Let us also note that for a system with n = 12 and 20, for which the angles between the various pins are not identical, when $\gamma > \gamma_{\min}$ the lower limit in expression (7) is a function of x, whose form is found from geometric considerations.

Analysis of the derived relationships shows that the solution of the problem reduces to the solution of a system of the two equations (4) and (5) with two unknown functions T(x) and $E^*_{\text{eff}}(x)$. If we introduce the following dimensionless variables:

$$\bar{T}=\frac{T}{T_0};\quad \bar{E}=\frac{E}{\sigma T_0^4};\quad \bar{r}_0=\frac{r_0}{L};\quad \bar{x}=\frac{x}{L};$$
$$\bar{z}=\frac{z}{L},\ \ldots,\ \bar{\xi}=\frac{\xi}{L}, \quad (12)$$

the above-derived relationships will become dimensionless and for the parameters of the problem for each variant of the spatial distribution of the pins we will have: the emissivity ε, the apex angle α of the pin, and the heat-conduction parameter

$$N=\frac{2\sigma T_0^3 L}{\lambda}. \quad (13)$$

By means of the derived relationships we can calculate the radiant power of five radiator variants from the number of pins and their spatial distribution (in accordance with the known regular polyhedrons).

System (4) and (5) was solved numerically on a computer by the method of successive approximations. The solution for the conical pins was found to be asymptotic for pins in the form of truncated cones as $L_1 \to L$ (see Fig. 1b).

The results of the solution are shown in Fig. 2 in the form of the relationship between the efficiency of the system and ε, α, and N for varying numbers of radiator pins. The efficiency of the system is understood to mean the ratio of the actual radiant flux to that limit flux which would be emitted by the system under consideration in the case of infinitely great thermal conductivity for the material of the pins and in the absence of radiant interaction between the pins:

$$Q_{\lim}=n\int_0^L 2\pi(L-x)\,\mathrm{tg}\,\frac{\alpha}{2}\,\sigma T_0^4\,dx. \quad (14)$$

The relationships in Fig. 2 make it possible to determine the flow of removed heat if the geometric dimensions of the system are known and if we also

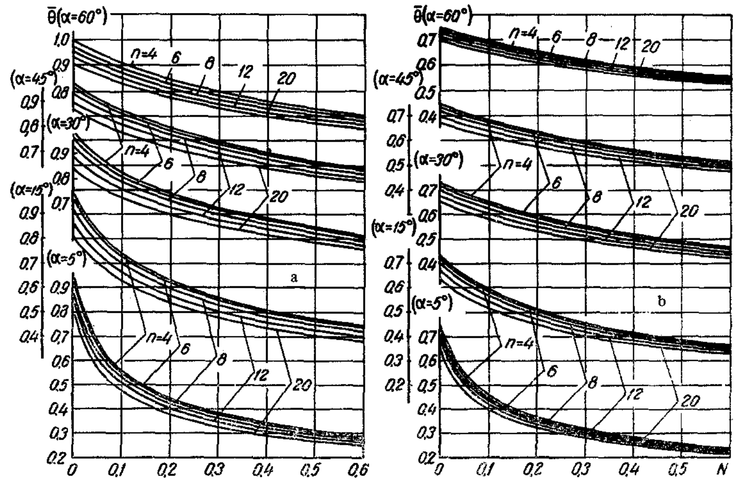

Fig. 2. Efficiency of radiating system versus α and N for various numbers of pins at ε = 1 (a) and 0.75 (b).

know the values of ε, T_0, and the thermal conductivity of the material (since in this case all of the parameters of the problem are known). The quantity $\bar{r}_0$ for each of the curves in Fig. 2 is determined from relationship (1), since a specific value of γ_{min} corresponds to each number n of the pins.

It is not difficult to verify that the above-derived results are valid also for hollow conical pins (Fig. 1d) in which there is no heat transfer between the inside surfaces and in which the wall thickness Δ varies along the pin axis so as to satisfy the following condition:

$$\frac{(L-x)\operatorname{tg}\frac{\alpha}{2}-\delta}{(L-x)\operatorname{tg}\frac{\alpha}{2}}=\varphi=\text{const}. \tag{15}$$

In this case, using the relationships in Fig. 2, we have to replace the parameter N by N_1 which is determined from the relationship

$$N_1=\frac{N}{1-\varphi^2}. \tag{16}$$

The relationships in Fig. 2 make it possible, in each specific case to optimize the radiator system being investigated.

This particular method of calculation can be extended also to systems in which the various groups of pins are situated under different conditions during the process of radiant interaction. In this case, the initial system of equations will include as many integrodifferential equations in the form of (4) and integral relationships in the form of (5) as there are groups of pins differing from each other, in terms of the heat-balance conditions.

NOTATION

Q is the heat flux along the pin at a distance x from its base; T_0 and T are the temperatures at the base of the pin and at a distance x from its base, respectively; α is the angle at the pin top; λ is the thermal conductivity; L and L_1 are the lengths of conical pin and of truncated cone-type pin, respectively; γ is the angle between the pin axes; σ is the Stefan-Boltzmann constant; ε is the emissivity of the pin surface; $E_{eff}(x)$ is the density of the effective radiation from the longitudinal section of the pin under consideration at a distance x from the pin base; $E_{eff}(z)$ is the same for the adjacent pin (Fig. 1b); $E_{inc\,z}(x)$ is the density of the incident radiation from the adjacent pin on the longitudinal section of the pin under consideration; $E^*_{inc}(x)$ is the same, averaged with respect to perimeter of the pin under consideration, $\Sigma E^*_{inc}(x)$ is the density of the incident radiation from all the other pins, averaged with respect to perimeter of the pin under consideration; $E^*_{eff}(x)$ is the density of the effective radiation calculated with account for incident radiation averaged with respect to the perimeter; r_0 is the radius of the body being cooled; δ is the wall thickness of the hollow pin; N is the pin's thermal conductivity; n is the number of pins in the radiator; γ_{min} is the minimum angle between pin axes for a given radiator; $\bar{\theta}$ is the efficiency of the radiating system.

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